

Electromagnetic Field Toroidal applications

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Introduction

According to *The Standard Handbook for Electrical Engineers*, by Fink and Beaty a toroidal coil is defined as “a simple magnetic circuit [that] is a uniformly wound torus ring”. By conducting a current, the winding then creates a relatively confined magnetic field within its core. The core can consist of various materials, including gaseous, to create different characteristics that fit the intentions of its design.

Having defined our focus of research, we will begin by analyzing the physical and mathematical derivation of the magnetic field produced by a toroid. The Biot-Savart and Ampere’s law will be used as our fundamental basis for this derivation. Hence allowing us to gain a correct understanding of how a toroidally confined magnetic field is created due to the geometry and flow of current in the winding. In addition, a comparison between the magnetic fields produced by toroidal and solenoid coils will be shown to observe the effects the winding geometry has on the magnetic field equations. Benefits of using a toroid vs. solenoid winding in specific circuitry design will also be compared. Finally, we will discuss the fundamentals of toroidal applications, mainly the most widely used Power transformers and then a relatively unknown topic, thermonuclear reaction stability.

Introduction of Magnetic Fields

Magnetism is associated with electrons spin and general movement in a current. Magnetic field causes a force on moving charges. Force, magnetic field, and velocity of moving charge are all perpendicular of each other. Magnetic force, F_m , cannot do work since it can only change the direction of moving charge. Magnetic field, also known as magnetic flux density or magnetic induction is symbolized as \mathbf{B} and is measured in Tesla (T). \mathbf{B} can be represented in field lines that always close on themselves, which explains why magnetic field across a closed surface vanishes, $\text{div } \mathbf{B}=0$.

The electric current loop is the most common source of \mathbf{B} . Magnetic field Intensity, \mathbf{H} , is related to Current density \mathbf{J} , in the following manner:

$$\mathbf{H} = \text{curl of } \mathbf{J} \text{ (I/m}^2\text{)} \quad (1)$$

$$\mathbf{J} = \int I \, dV \quad \text{where } V = \text{volume, } I = \text{current} \quad (2)$$

$$I = \int (\mathbf{H} \cdot d\mathbf{l}) \text{ over closed surface} \quad (3)$$

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B relates to **H**:

$$\mathbf{H} = \mathbf{B}\mu_0 \text{ (A/m}^2\text{)} \quad (4)$$

Electric currents have a magnetic dipole moment equal to Ia , current times loop area. Particle with magnetic dipole moments are known as magnetic dipoles, such as in the case of a magnet. When subject to external magnetic force, magnetic dipoles may experience torque that aligns their poles to external magnetic forces.

All matter has a magnetic susceptibility, which is the property that shows degree of susceptibility of that matter to become magnetized due to an external \mathbf{B} field. Some materials such as iron have strong magnetization properties due to alignment of magnetic moments within small regions called domains. Some alloys like Neodymium keep their domains permanently aligned and serve to make strong magnets. Heating a magnet to high temperatures may destroy its magnetic properties since the alignments causing them are destroyed.

Derivation of Magnetic Field Equations from the Biot-Savart Law

[As derived by David J Griffith's book, Introduction to Electrodynamics]

\mathbf{B} lines form loops around current. According to Biot-Savart law:

$$\mathbf{B} = \int \frac{\mu_0 I d\mathbf{l} \times \hat{\mathbf{r}}}{4\pi |r|^2}, \quad (\text{Wb/m}^2) \quad (5)$$

" I " represent steady current across a length of wire,

$d\mathbf{l}$ representing a partial of the total length, over square distance, in the direction perpendicular to direction of current.

μ_0 is equal to permeability of free space equal to $4\pi \times 10^{-7} \text{ N/A}^2$.

$\hat{\mathbf{r}}$ is the displacement unit vector,

while $\mathbf{r} = r\hat{\mathbf{r}}$ is the displacement vector in the direction from the wire toward point where \mathbf{B} is computed.

\mathbf{B} can then be seen as:

$$\mathbf{B} = \int \frac{\mu_0 I d\mathbf{l} \times \mathbf{r}}{4\pi |r|^3}, \quad (\text{Wb/m}^2) \quad (6)$$

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Magnitude of B is at its max when angle between dl and \hat{r} is 90° . B in a current loop is obtained by summing individual partial contributions of all segments of the circuit. For the long wire the formula can be simplified to:

$$B = \mu_0 I / (2\pi r) \Phi \quad (\text{Wb/m}^2) \quad (7)$$

Φ is the unit vector around electric current on a wire.

Solenoid Magnetic field as given by the Book Fundamentals of Applied Electrodynamics by Ullaby, Michielssen, and Ravaioli

Magnetic field at a distance r of electric current loop is obtained through the equation (5.89):

$$B = \mu_0 I N / (\sin(\Phi_1) - \sin(\Phi_2)) \quad (8)$$

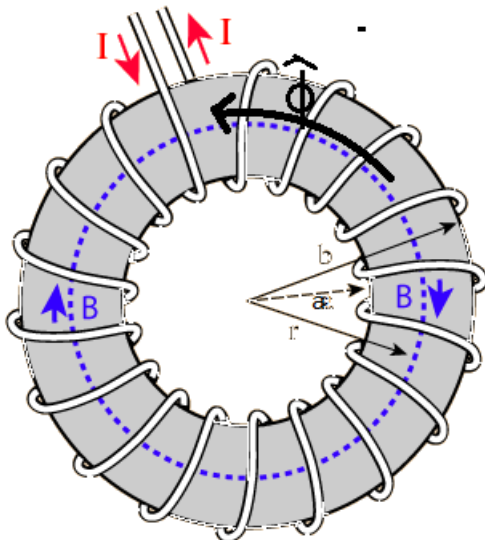
In the central part of a long solenoid far from its ends, $\Phi_2 = -\Phi_1 = |\pi/2|$ (9)

If the inside of solenoid is empty the equation approximates:

$$B = \mu_0 I n = \mu_0 I N / L \quad (\text{Wb/m}^2) \quad (10)$$

In this case n is number of turns per length of solenoid.

Comparison of Toroid with Solenoid Field Equations:



Hence, from observation the formula for a toroid approximates that for the long solenoid, the only difference being that L is now instead given by the circumference of a toroidal axis.

$$B = \mu_0 I N / L = \mu_0 I N / 2\pi r \quad (\text{Wb/m}^2) \quad (11)$$

This can be further proven:

By the right hand rule, the B is in $-\Phi$, so $B = -\Phi B$. The total current passing through the contours NI with direction into the page. According to right hand rule, current should point toward four fingers when thumb points into \hat{O} . Since Current goes in instead of out, total current is negative.

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$$\mathbf{I} = \mu_o \int (\mathbf{B} * d\mathbf{l}) \text{ over closed surface} = \mu_o \int (-\Phi \mathbf{B}) * \Phi r d\Phi, 0 < \Phi < 2\pi, = -2 \pi r B \mu_o = -NI \quad (12)$$

$$B = NI / (2 \pi r) \quad (\text{Wb/m}^2) \quad (13)$$

$$\mathbf{B} = -\Phi NI / (2 \pi r) \quad (\text{Wb/m}^2) \quad (14)$$

Magnetic flux Φ integration of \mathbf{B} over a surface, and its units are Wb

In a solenoid with its axis on z-axis

$$\Phi = \int (\mathbf{z}(\mu NI/l) * \mathbf{z} ds) = \mu ANI/l \quad (\text{Wb}) \quad (15)$$

A being the cross sectional area of the loop.

The equation for the solenoid is practically the same except for L being $2 \pi r$

$$\Phi = \mu SNI / (2 \pi r) \quad (\text{Wb}) \quad (16)$$

Fundamentals of Toroidal Inductor Transformer use in Power Distribution

(Fundamentals of applied, 201)

Self-inductance, ratio of magnetic flux to \mathbf{I} is given by:

$$\mathbf{L} = \Phi / \mathbf{I} \quad (\text{H} = \text{Wb/A}) \quad (17)$$

Magnetic flux linkage Λ is total magnetic linking flux in a circuit. In a solenoid and toroid with N turns Λ is the flux linking all loops of structure:

$$\Lambda = N\Phi = \mu SIN^2 / (2 \pi r) \quad (\text{Wb}) \quad (18)$$

For a solenoid and toroid:

$$\mathbf{L} = \Phi / \mathbf{I} = \Lambda / \mathbf{I} = \mu SN^2 / (2 \pi r) \quad (\text{H} = \text{Wb/A}) \quad (19)$$

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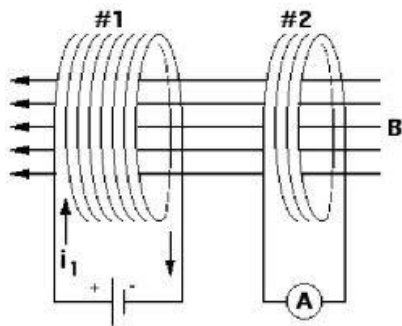


Fig of magnetic flux inducing current retrieved from http://www.astarmathsandphysics.com/a_level_physics_notes/electricity/a_level_physics_notes_self_inductance_and_mutual_inductance.html (Smith, 2011)

Magnetic coupling between two conducting structures is described through mutual inductance.

Power supply forces a current in loop one, which causes a magnetic field through the middle of the loop resulting in Φ_{12} passing through loop 2.

$$\Phi_{12} = \int (\mathbf{B} \cdot d\mathbf{s}) \text{ over } S_2 \quad (\text{Wb}) \quad \cdot = \text{dot product} \quad (19)$$

Then total magnetic flux linkage through loop 2 is:

$$\Lambda_{12} = N_2 \Phi_{12} = N_2 \int (\mathbf{B} \cdot d\mathbf{s}) \text{ over } S_2 \quad (\text{Wb}) \quad (20)$$

And mutual inductance is obtained by using magnetic flux linkage:

$$\mathbf{L}_{12} = \Lambda_{12} / \mathbf{I}_1 = (N_2 / \mathbf{I}_1) \int (\mathbf{B} \cdot d\mathbf{s}) \text{ over } S_2 \quad (\text{H} = \text{Wb/A}) \quad (21)$$

This mutual inductance configuration is how transformers are basically made.

Magnetic energy is obtained as follows:

$$\mathbf{W}_m = \int (P \, dt) = \int (v_i \, dt) = L \int (i \, di) = LI^2 / 2 \quad (22)$$

In transmission lines AC power is much more efficient and easier to make thanks through transformers ability to produce V with change in magnetic flux. AC current, which travels as sinusoidal waves, produce AC V produced by the changing flux.

$$\mathbf{V}_{emf} = -N \, d \Phi / dt = -N \, d [\int (\mathbf{B} \cdot d\mathbf{s}) \text{ over } S_2] / dt \quad (23)$$

Or more specifically transformer emf Voltage:

$$\mathbf{V}_{tr \, emf} = -N \int (d\mathbf{B} / dt \cdot d\mathbf{s}) \quad (24)$$

(Nave , 2006)

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Toroidal applications in Thermonuclear Stabilization

An elegant and powerful use of toroids is found in thermonuclear fusion research. It is of greatest interest to always maintain constant control of nuclear reactions. By general, nuclear energy researchers have developed two means of confining and controlling the reactions.

- Inertial Confinement
- Magnetic Confinement

Under the tremendous amounts of heat produced by the reaction, there exists no material that can withstand such elevated temperatures and serve as successful containers to encase the tremendous amounts of energy produced. Hence researchers use these two methods, either inertial or magnetic confinement. It is of our interest to further study the use of magnetic confinement due to the central use of the toroid in this method of thermonuclear reaction encasement.

Tokamak- Function Fundamentals

As stated in The Great Soviet Encyclopedia, the Tokamak was first developed in the Kurchatov Institute of Moscow in the 1950's [reference]. This machine after much research and development was built in 1968 and proved successful as a means of creating a reaction and encasing the hot plasma produced by such, through creation of magnetic field walls. Nowadays, the Tokamak is one of the most common machines used for nuclear reaction research, amongst others such as Spheromak and the Stellarator. Our main focus will be the Tokamak.

Having an enclosed toroidal geometry, an electric field is produced within the cavity through outer magnetic transformer cores. As the fuel inside the cavity is hydrogen isotopes such as deuterium and tritium, the flow of this electrical field creates plasma and elevates the temperature within. Other forms of energy are also used to help elevate the temperature within to that needed for fusion reaction, such as radio frequency heating and injection of high-energy particle beams.

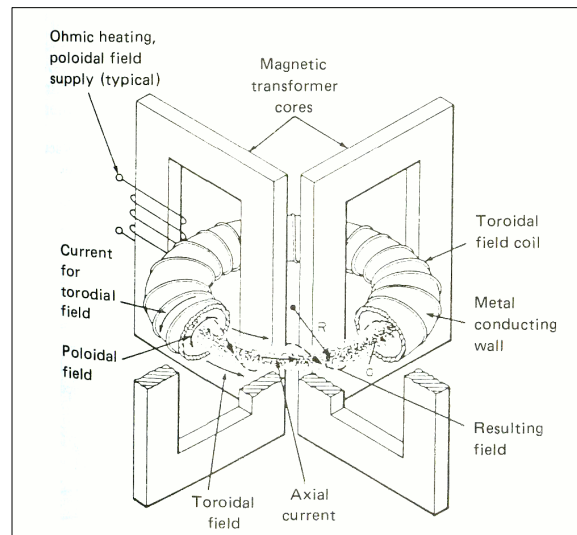


Figure 2-A Tokamak Diagram [reference standard handbookEE]

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By simply observing the Tokamak diagram [Figure2-A], one may at first not understand how it is that an electric field is created by the outer magnetic transformer cores. However, by noting that these cores are wrapped by a current carrying coil, a

changing magnetic field is produced, that rotates around the enclosing magnetic transformer cores.

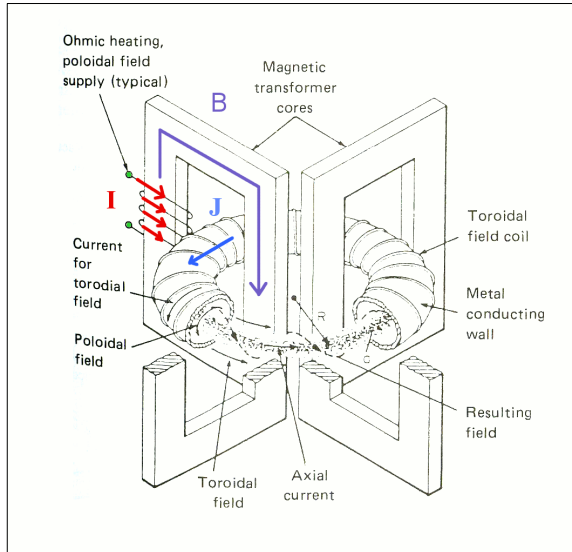


Figure 2-B Current Produced by transformer cores [reference standard handbookEE]

One can note the use of Ampere's law with Maxwell's correction (generally known as Maxwell's fourth equation)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (25)$$

[Reference: Intro to Electrodynamics]

Such rotating B field enclosed by the transformers, through observation definitely has a non-zero curl. Thus a non-zero current density J is produced at the central part enclosed by the magnetic transformer

The elevated temperatures of over a 100 million degrees kelvin, (approximately 50 times hotter than the inner core of the sun!) allows the plasma to produce a nuclear reaction, and therefore as most explosions tends to explode in an outwardly fashion. However, to prevent this outward explosion that could not only prove to be deadly but also highly inefficient in terms of energy production as the plasma would quickly cool down and stop the fusion. Magnetic fields are used to control such.

Magnetic Confinement - Field Production

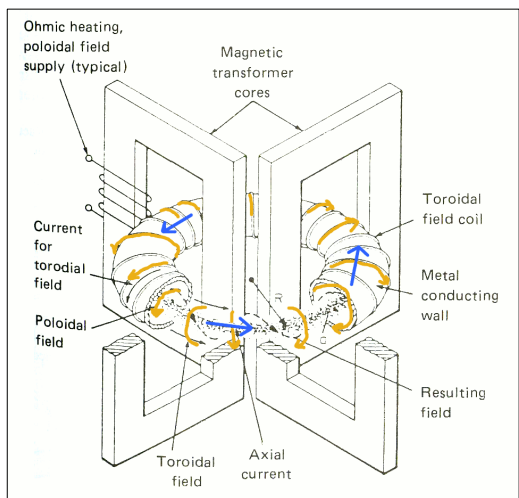
From the current produced by the magnetic transformer cores, this electric current produces a rotating magnetic field that compresses the plasma. [Figure 2-B]

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

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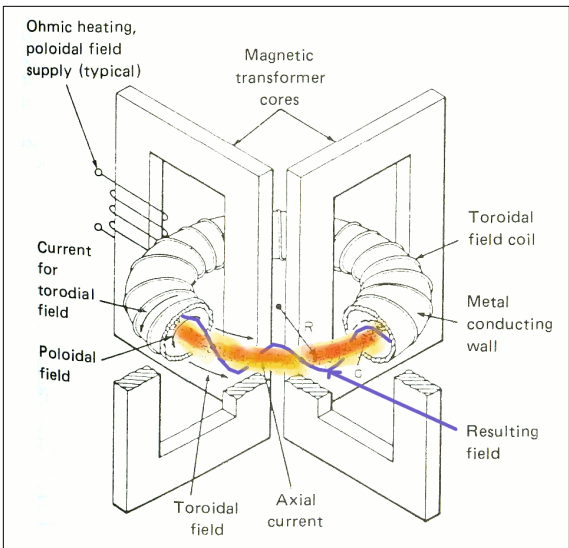
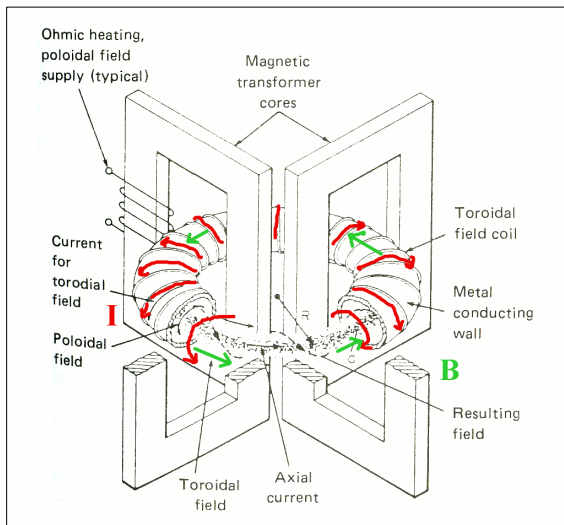
Faraday’s law and Ampere’s law can be used to see this interaction and production of the field from the rotating current.

Nonetheless, this magnetic field is insufficient to properly confine the plasma within and is further aided by a separately created magnetic field.

Figure 2-C Magnetic field shown in yellow produced by inner circulating plasma current [reference standard handbookEE]

Like previously studied toroid inductors, the toroid chamber is wound with current carrying conductors that in turn produce an axial magnetic current to “flow” within the chamber.

Figure 2-D Magnetic field shown in green produced by toroidal currents [reference standard handbookEE]



The combination of both of these magnetic fields thus creates a helically rotating magnetic field that can successfully encase and compress

Figure 2-E Final resulting magnetic field results in helical shape due to vector addition of poloidal and toroidal fields [reference standard handbookEE]

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Conclusion

Through observation of the various derivations of the magnetic fields produced by toroids and also through comparison with a long solenoid, a greater understanding of the functions of toroids was achieved. Furthermore, two applications for toroids were chosen for a greater depth of study. Transformers were investigated for their wide use in power distribution as these allow for voltage stepping due to the efficiency of high voltage transmission and the necessity of having a standard voltage at user reception. Finally analysis and the fundamentals of function of Tokamak were studied for their applications of the magnetic field created by toroidal coils.

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